AXISYMMETRIC SPHERICAL ELECTROMAGNETIC WAVES AND CAVITY RESONATORS ON THEIR BASE

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ABSTRACT

The solution of a problem of propagation of the standing spherical electromagnetic waves in space, free from sources, in case of isotropic medium on assumption of rotational symmetry, is offered by exact solution of the Maxwell uniform equations in a spherical coordinate system. Analytical expressions for components of fields of the standing symmetric spherical E- and H-waves and equations of lines of force for different modes are obtained. The graphic pictures of lines of force, which reflect the structures of fields of the standing spherical E- and H-waves, are represented. The cavity resonators, formed by conducting spheres that envelop a cavity of the resonator in absence or in presence at the centre of resonator "core", are considered. The own values and resonators' own functions that allow to determine sets of their resonant frequencies and appropriate structures of fields are found.

Keywords: standing electromagnetic waves, spherical coordinates, isotropic medium, cavity resonators, convergence in point, divergence from point.

1. INTRODUCTION

The interest to the models spherical electromagnetic waves arises in the theory of radio propagation, in the analysis of UHF spherical resonators, in optics, in problems of laser initiation of reaction of thermonuclear synthesis and etc. Within the work – Ref.1, submitted by the authors early, the problem of propagation of a spherical electromagnetic wave in space, free from sources is considered in new statement for case of an isotropic medium and in the supposition of a rotational symmetry. Thus as initial expressions the uniform equations of the Maxwell concerning components of vectors of strengths of electrical and magnetic fields noted in spherical coordinates are used. As a result the detailed analytical expressions based on a strict solution of a system of initial equations, for all fields components of travelling spherical E- and H-waves are obtained. Thus, the described in work – Ref.1 approach to a solution of the this problem allows directly, without use of Hertz's vector, accepted in an electrodynamics, to obtain analytical expressions that define by themselves the new base, in the mathematical plan, of spherical functions that reflect electromagnetic waves propagating to the centre and from the centre of spherical coordinates.

The aim of present work is, at first, consideration of the problem of propagation of standing spherical electromagnetic wave in space, free from sources is presented for case of an isotropic medium and in the supposition of a rotational symmetry. The converging and diverging spherical waves were considered - Ref.1 as independent, now they are represented transformation by one in another at transiting through the centre or at reflection from ideally conducting "core" which is situated at the centre and aspire into point. Both these situations create in each of two cases for E and H-waves two different systems of spherical functions already in the form of standing waves. The analytical expressions, defining components standing spherical E- and H-waves, and also equations

of lines of force for different types of oscillations are obtained at presence of "core" at the centre and without it. The numerical solution of these equations has allowed to calculate and to present graphically pictures of lines of force, defining structures, appropriate to them, of fields of standing spherical E- and H-waves. Secondly, the authors state a task to define own values and own functions for the cavity resonators formed by conducting spheres that envelop a cavity of the resonator in absence or in presence at the centre "core". Solution of this task allows to determine sets of the resonant frequencies.

2. STANDING SPHERICAL ELECTROMAGNETIC WAVES

Originally, we consider connection between amplitudes of converging and diverging spherical electromagnetic waves, which are independent of radial and angular coordinates, at presence of "core", that is situated at the centre and aspires into point. For this purpose we present the "core" as a conducting sphere of finite value of radius r_0. On a surface of sphere the Leontovich's boundary condition should be fulfilled at $r = r_0$, for E-wave:

$$E_\varphi^+ + E_\varphi^- = 0 \tag{1}$$

and accordingly for H-wave:

$$E_\theta^+ + E_\theta^- = 0 . \tag{2}$$

As a result of substitution of the appropriate fields components of E- and H-waves in a condition of Leontovich, when radius of sphere r_0 is aspiring to zero, the expressions (1) and (2) for odd values n (n characterises number of lobes of the angular diagram of amplitude distribution of spherical waves on a latitude – Ref.1) assume the form:

$$E_\varphi^{0+} = E_\varphi^{0-} = E_\varphi^0 , \tag{3}$$

$$H_\varphi^{0+} = H_\varphi^{0-} = H_\varphi^0 , \tag{4}$$

where E_φ^0 and H_φ^0 - general conditional labels for E-mode and H-mode accordingly.

For even values n we have:

$$E_\varphi^{0+} = -E_\varphi^{0-} = E_\varphi^0 , \tag{5}$$

$$H_\varphi^{0+} = -H_\varphi^{0-} = H_\varphi^0 \tag{6}$$

Let's note, that in a case, when at the centre of a considered coordinate system there is no "core", the conditions of connection between independent amplitudes for even and odd n values vary on the conditions opposite in sign, in comparison with a case at presence of "core". It follows from a condition of preservation of a direction of a vector of polarisation vector for converging and diverging spherical waves.

Further, according to the obtained conditions we present fields components of standing *rEnH*ical E- and H-waves in cases of presence and absence of "core" for different values n.

1) There is "core", $n=1$:

$$\left\{\begin{matrix} E_\varphi \\ H_\varphi \end{matrix}\right\} \sim \left[\frac{Cos(kr)}{kr} - \frac{Sin(kr)}{(kr)^2}\right] Sin\theta \exp(j\omega t)\,; \tag{7}$$

$$\left\{\begin{matrix} H_r \\ E_r \end{matrix}\right\} \sim 2(\pm j)\left[\frac{Cos(kr)}{(kr)^2} - \frac{Sin(kr)}{(kr)^3}\right] Cos\theta \exp(j\omega t)\,; \tag{8}$$

$$\left\{\begin{matrix} H_\theta \\ E_\theta \end{matrix}\right\} \sim (\mp j)\left[\frac{Sin(kr)}{kr} + \frac{Cos(kr)}{(kr)^2} - \frac{Sin(kr)}{(kr)^3}\right] Sin\theta \exp(j\omega t). \tag{9}$$

For $n=2$:

$$\left\{\begin{matrix} E_\varphi \\ H_\varphi \end{matrix}\right\} \sim \left[\frac{Sin(kr)}{kr} + \frac{3Cos(kr)}{(kr)^2} - \frac{3Sin(kr)}{(kr)^3}\right] Sin(2\theta) \exp(j\omega t); \tag{10}$$

$$\left\{\begin{matrix} H_r \\ E_r \end{matrix}\right\} \sim 2(\pm j)\left[\frac{Sin(kr)}{(kr)^2} + \frac{3Cos(kr)}{(kr)^3} - \frac{3Sin(kr)}{(kr)^4}\right]\cdot \left(3Cos^2\theta - 1\right)\exp(j\omega t)\,; \tag{11}$$

$$\left\{\begin{matrix} H_\theta \\ E_\theta \end{matrix}\right\} \sim (\mp j)\left[\frac{Cos(kr)}{kr} - \frac{3Sin(kr)}{(kr)^2} - \frac{6Cos(kr)}{(kr)^3} + \frac{6Sin(kr)}{(kr)^4}\right]\cdot Sin(2\theta) \exp(j\omega t). \tag{12}$$

For $n=3$:

$$\left\{\begin{matrix} E_\varphi \\ H_\varphi \end{matrix}\right\} \sim \left[\frac{Cos(kr)}{kr} - \frac{6Sin(kr)}{(kr)^2} - \frac{15Cos(kr)}{(kr)^3} + \frac{15Sin(kr)}{(kr)^4}\right]\cdot Sin\theta\left(5Cos^2\theta - 1\right)\exp(j\omega t)\,; \tag{13}$$

$$\left\{\begin{matrix} H_r \\ E_r \end{matrix}\right\} \sim 2(\pm j)\left[\frac{Cos(kr)}{(kr)^2} - \frac{6Sin(kr)}{(kr)^3} - \frac{15Cos(kr)}{(kr)^4} + \frac{15Sin(kr)}{(kr)^5}\right]\cdot Cos\theta\left(5Cos(2\theta) - 1\right)\exp(j\omega t)\,; \tag{14}$$

$$\left\{\begin{matrix} H_\theta \\ E_\theta \end{matrix}\right\} \sim (\pm j)\left[\frac{Sin(kr)}{kr} + \frac{6Cos(kr)}{(kr)^2} - \frac{21Sin(kr)}{(kr)^3} - \frac{45Cos(kr)}{(kr)^4} + \frac{45Sin(kr)}{(kr)^5}\right] Sin\theta\left(5Cos^2\theta - 1\right)\exp(j\omega t)\,. \tag{15}$$

2) "Core" is absent, $n=1$:

$$\left\{\begin{matrix} E_\varphi \\ H_\varphi \end{matrix}\right\} \sim j\left[\frac{Sin(kr)}{kr} + \frac{Cos(kr)}{(kr)^2}\right] Sin\theta \exp(j\omega t)\,; \tag{16}$$

$$\left\{\begin{matrix} H_r \\ E_r \end{matrix}\right\} \sim \mp 2\left[\frac{Sin(kr)}{(kr)^2} + \frac{Cos(kr)}{(kr)^3}\right] Cos\theta \exp(j\omega t)\,; \tag{17}$$

$$\left\{\begin{matrix} H_\theta \\ E_\theta \end{matrix}\right\} \sim \pm\left[\frac{Cos(kr)}{kr} - \frac{Sin(kr)}{(kr)^2} - \frac{Cos(kr)}{(kr)^3}\right] Sin\theta \exp(j\omega t). \tag{18}$$

For $n=2$:

$$\left\{\begin{matrix} E_\varphi \\ H_\varphi \end{matrix}\right\} \sim j\left[\frac{Cos(kr)}{kr} - \frac{3Sin(kr)}{(kr)^2} - \frac{3Cos(kr)}{(kr)^3}\right] Sin(2\theta) \exp(j\omega t); \tag{19}$$

$$\left\{\begin{matrix} H_r \\ E_r \end{matrix}\right\} \sim \mp 2\left[\frac{Cos(kr)}{(kr)^2} - \frac{3Sin(kr)}{(kr)^3} - \frac{3Cos(kr)}{(kr)^4}\right]\cdot \left(3Cos^2\theta - 1\right)\exp(j\omega t); \tag{20}$$

$$\left\{\begin{matrix} H_\theta \\ E_\theta \end{matrix}\right\} \sim \mp\left[\frac{Sin(kr)}{kr} + \frac{3Cos(kr)}{(kr)^2} - \frac{6Sin(kr)}{(kr)^3} - \frac{6Cos(kr)}{(kr)^4}\right]\cdot Sin(2\theta) \exp(j\omega t). \tag{21}$$

For $n=3$:

$$\left\{\begin{matrix} E_\varphi \\ H_\varphi \end{matrix}\right\} \sim j\left[\frac{Sin(kr)}{kr} + \frac{6Cos(kr)}{(kr)^2} - \frac{15Sin(kr)}{(kr)^3} - \frac{15Cos(kr)}{(kr)^4}\right]\cdot Sin\theta\left(5Cos^2\theta - 1\right)\exp(j\omega t)\,; \tag{22}$$

$$\left\{\begin{matrix} H_r \\ E_r \end{matrix}\right\} \sim \mp 2\left[\frac{Sin(kr)}{(kr)^2} + \frac{6Cos(kr)}{(kr)^3} - \frac{15Sin(kr)}{(kr)^4} - \frac{15Cos(kr)}{(kr)^5}\right]\cdot Cos\theta\left(5Cos(2\theta) - 1\right)\exp(j\omega t)\,; \tag{23}$$

$$\left\{\begin{matrix} H_\theta \\ E_\theta \end{matrix}\right\} \sim \pm\left[\frac{Cos(kr)}{kr} - \frac{6Sin(kr)}{(kr)^2} - \frac{21Cos(kr)}{(kr)^3} + \frac{45Sin(kr)}{(kr)^4} + \frac{45Cos(kr)}{(kr)^5}\right] Sin\theta\left(5Cos^2\theta - 1\right)\exp(j\omega t)\,. \tag{24}$$

L*H*'r analy*rH*b*H*aviour of am*E*litud*H* of E- and H-waves at the centre. In the first case (at presence of "core") the fields am*E*litud*H* of *rEnH*ical E- and H-waves accept finite values at the centre. In the second case ("core" is absent) the singularity as aspiring of amplitudes of oscillating fields to infinity is formed.

3. STRUCTURA OF STANDING SPHERICAL ELECTROMAGNETIC WAVES FIELDS

The pictures of lines of force of electrical and magnetic fields for different modes (for cases of presence and absence of "core" at the centre) are especially interesting. The general form the equation of electrical lines of force in a spherical coordinate system looks like:

$$\frac{dr}{E_r} = \frac{rd\theta}{E_\theta} = \frac{rSin\theta\, d\varphi}{E_\varphi} \tag{25}$$

and, accordingly, equation of magnetic lines of force:

$$\frac{dr}{H_r} = \frac{rd\theta}{H_\theta} = \frac{rSin\theta\, d\varphi}{H_\varphi}. \tag{26}$$

We obtain individual equations of lines of force according to presented expressions for the appropriate fields com*E*on*H*ntr, for *H*kam*E*l*H*, for E-mode at different values n.

1) TE-mode, there is "core", $n=1$.

In this case, equation (26) assume the form:

$$H_\theta dr = rH_r d\theta\,. \tag{27}$$

As a result of substitution of the expressions (8), (9) for components E_r and E_θ into equation (27) we obtain ordinary differential equation of the first order with dividing variables. The solving of integral of left and right parts of equation with help of program "Mathematics 5.0" gives us general solution of this differential equation. After not complicated transformations we obtain the equation of magnetic lines of force for $n = 1$:

$$\left| H\mathrm{al}\,(\mathrm{c}r) - \frac{\mathrm{r\,i}n(\mathrm{c}r)}{\mathrm{c}r} \right| \cdot \mathrm{r\,i}n^2\theta = H^*, \qquad (28)$$

where * - constant of integration defining a set of lines of force.

Equation of magnetic lines of force for $n = 2$:

$$\frac{\left|3\mathrm{c}rH\mathrm{al}\,(\mathrm{c}r) + ((\mathrm{c}r)^2 - 3)\mathrm{r\,i}n(\mathrm{c}r)\right|}{(\mathrm{c}r)^2}\,\mathrm{r\,i}n^2\theta \left|H\mathrm{al}\,\theta\right| = H^* \qquad (29)$$

and also for $n = 3$:

$$\frac{\left|\mathrm{c}r((\mathrm{c}r)^2 - 15)H\mathrm{al}\,(\mathrm{c}r) + 3(5 - 2(\mathrm{c}r)^2)\mathrm{r\,i}n(\mathrm{c}r)\right|}{\left|\mathrm{c}r\right|^3} \cdot$$

$$\cdot \left|3 + 5H\mathrm{al}\,(2\theta)\right| \mathrm{r\,i}n^2\theta = H^*. \qquad (30)$$

2) TE-wave, "core" is absent.

The equation defining magnetic lines of force for $n = 1$:

$$\left| \mathrm{r\,i}n(\mathrm{c}r) + \frac{H\mathrm{al}\,(\mathrm{c}r)}{\mathrm{c}r} \right| \mathrm{r\,i}n^2\theta = H^*; \qquad (31)$$

for $n = 2$:

$$\frac{\left|((\mathrm{c}r)^2 - 3)H\mathrm{al}\,(\mathrm{c}r) - 3\mathrm{c}r\mathrm{r\,i}n(\mathrm{c}r)\right|}{(\mathrm{c}r)^2}\,\mathrm{r\,i}n^2\theta \left|H\mathrm{al}\,\theta\right| = H^*; \qquad (32)$$

for $n = 3$:

$$\frac{\left|3(2(\mathrm{c}r)^2 - 5)H\mathrm{al}(\mathrm{c}r) + \mathrm{c}r((\mathrm{c}r)^2 - 15)\mathrm{r\,i}n(\mathrm{c}r)\right|}{\left|\mathrm{c}r\right|^3} \cdot$$

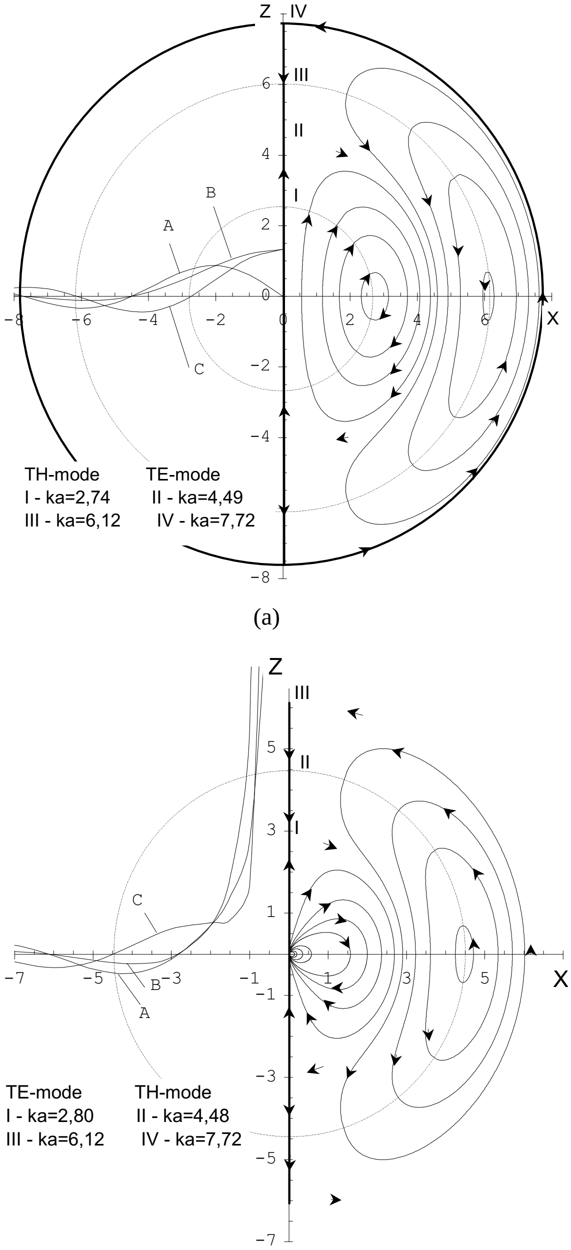
$$\cdot \left|3 + 5H\mathrm{al}\,(2\theta)\right| \mathrm{r\,i}n^2\theta = H^*. \qquad (33)$$

It is necessary to note that at change mode fields E and H are interchanged the position, i.e. the equations of magnetic lines of force for E-mode become the equations of electrical lines of force for H-mode at each concrete value n. Lines of force of electrical field for TE-mode and also magnetic lines of force for TH-mode are concentric circles situated in planes which are parallel equatorial. Thus, the obtained equations of lines of force make possible visualisation of a set of lines of force of these fields.

According to results of numerical solving of these equations, the construction of pictures of lines of force reflecting structure of fields of standing spherical electromagnetic waves at presence of "core" at the centre and without it, for different values n, is carried out. In fig.1(a) in the right half-plane the set of lines of force is represented as curls, which are magnetic for TE-mode and electrical for TH-mode for case of presence of "core" at the centre and $n = 1$. The curves of dependence of normalised amplitudes of components of standing TE- and TH-wave from normalised coordinate $\mathrm{c}r$ at constant value θ is represented in the left half-plane. From this graph follows, that the amplitudes of fields accept final values at the centre. The similar picture is submitted in fig.1(b) for a case, when there is no "core" at the centre. From this picture follows that the amplitudes of oscillating fields in an approximation to the centre essentially grow and asymptotically aspire to infinity at the centre. This situation was excluded in works of other authors because it ostensibly has not physical sense. The given representation allows to consider it as objective reality.

Cases with the "core" at the centre of coordinate system for $n = 2$ and $n = 3$ is represented in the figs. 2(a) and 3(a) accordingly. The situations, when at the centre the "core" is absent for $n = 2$ and $n = 3$, are submitted also in figs.2(b) and 3(b) accordingly.

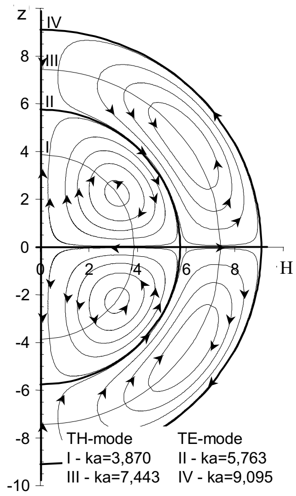
Let's note, that owing to a symmetry on azimuth coordinate φ, the pictures of lines of force are symmetric for areas $0 < \theta < \pi$ and $\pi > \theta > 2\pi$. For cases $n > 1$, there is the separation of curls on the latitude according to a value n.



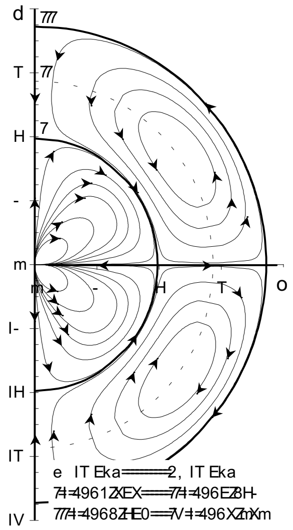
(a)

(b)

Fig.1. The pictures of fields in meridian cut for n = 1; (a) shows the standing wave in case of presence of conducting point "core" at centre; (b) – the same in case of absence "core" at centre. Here A - E_φ or E_φ; B - E_r or E_r; C - E_θ or E_θ.



(a)



(b)

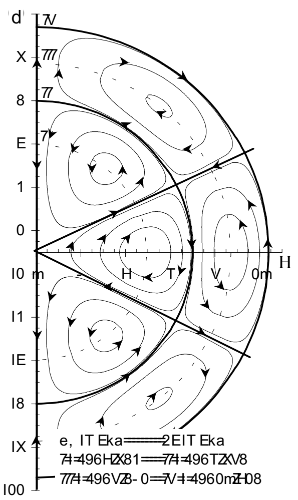
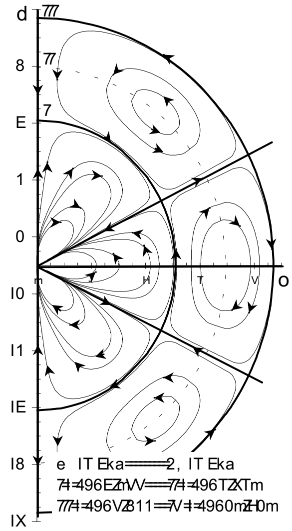


Fig.2. The similar pictures of fields for n = 2 also in case of "core" presence (a) and "core" absence (b).

(a)



(b)

Fig.3. The similar pictures of fields for n=3.

4. SPHERICAL RESONATORS WITH A ROTATIONAL SYMMETRY OF MODES

The concept of "an electrical wall ", accepted in an electrodynamics, allows to proceed to the analysis of spherical resonators with a rotational symmetry of modes. Let's volumetric resonators formed by conducting spheres, enveloping a concavity of the resonator at presence of "core" at the centre or without it. On a surface of the resonator the Leontovich's boundary condition should be fulfilled:

$$\text{TEIt -wave } H_{\varphi} 2nE^{*}\Big|_{E=r} = 1\,; \tag{34}$$

$$\text{TEIti -wave } \mathrm{r}_{\theta} 2nE^{*}\Big|_{E=r} = 1\,, \tag{35}$$

where r - radius of resonator. Further, according to this condition, the dispersing equations for TE- and TH-modes are obtained. The own values, which are solutions of the appropriate dispersing equations, and snet Ec at Tler yeaohegt ETf snet mdt fal t Hi t The.l gt ETf snet resonators with "core" and without it are calculated for different values i .They are shown at figures.

5. CONCLUSION

In this paper, the standing axisymmetric spherical electromagnetic waves are analysed as waves, travelling to centre, passing through it or reflecting from conducting point "core" at centre. The analysis is based on the analytical expressions what were earlier derived by authors with help of exact solutions of Maxwell uniform equations in spherical coordinates. These solutions are not commonly known but theirs validity is clearness from presented here field pictures. The peculiarity of the field behaviour near the centre in case of absence of conducting "core" at centre is unconditional novelty.

6. REFERENCES

1. Yuri A. Zyuryukin, Mariya V. Pavlova, "Axially symmetric spherical electromagnetic waves in isotropic medium" // PROCEEDINGS OF SPIE; Saratov Fall Meeting 2000, Coherent Optics of Ordered and Random Media; USA; Vol. 4242, pp. 53-58.